

INTRODUCTION :

Since second class at school, I was fascinated by Nth degree equations, which can't be solved at present even by the most advanced mathematicians, but for which there exists (It has been demonstrated), n real solutions :

$$x_1, x_2, \dots, x_n$$

I discovered by chance an unknown theorem which never appears in any mathematical book. I demonstrate this theorem, for equations of degree 1, 2 et 3, (see below).

This theorem binds the derivatives of function :

$$y = a_0 x^n + a_1 x^{n-1} + \dots a_n$$

To his roots $x_1, x_2, \dots, x_n$

That is to say the values : $y'_1, y'_2, \dots, y'_n$

This theorem can be written as :

THEOREM :

The product of derivatives at the roots of Nth degree equations is equal to the discriminant of this equation :

$$y'_1 y'_2 \dots y'_n = \Delta$$

And also equal to the product of the roots difference, taken 2 by 2.

Example for a second degree equation :

Let x_1 and x_2 the equation roots :

$$y = ax^2 + bx + c$$

$$y'_1 = 2ax_1 + b$$

$$y'_2 = 2ax_2 + b$$

$$y'_1 y'_2 = (2ax_1 + b)(2ax_2 + b)$$

$$y'_1 y'_2 = 4a^2 x_1 x_2 + 2ab(x_1 + x_2) + b^2$$

$$y'_1 y'_2 = 4a^2 \frac{c}{a} + 2ab \left(-\frac{b}{a} \right) + b^2$$

$$y'_1 y'_2 = 4ac + 2ab \left(-\frac{b}{a} \right) + b^2$$

$$y'_1 y'_2 = 4ac - b^2$$

$$y'_1 y'_2 = (x_1 - x_2)(x_2 - x_1) = \Delta$$

Consequences : It is possible to compute the PI number, knowing that all his root derivatives are equal to + or - 1.

CASE of DEGREE 3 EQUATION :

Let us consider the following equation :

$$y = x^3 + px + q$$

Whose roots are x_1, x_2, x_3 :

$$y' = 3x^2 + p$$

$$y'_1 = 3x_1^2 + p \quad y'_2 = 3x_2^2 + p \quad y'_3 = 3x_3^2 + p$$

$$y'_1 y'_2 y'_3 = (3x_1^2 + p)(3x_2^2 + p)(3x_3^2 + p)$$

$$y'_1 y'_2 y'_3 = 9(x_1 x_2 x_3)^2 + 3p^2 x_1^2 x_2^2 x_3^2 + 9p(x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_1^2) + p^3$$

After development, and assuming that :

$$x_1 + x_2 + x_3 = 0$$

$$x_1 x_2 + x_2 x_3 + x_3 x_1 = p$$

$$x_1 x_2 x_3 = -q$$

$$y'_1 y'_2 y'_3 = 27(-q)^2 + 3p^2(-2p) + 9p(x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_1^2) + p^3$$

When developped :

$$(x_1 x_2 + x_2 x_3 + x_3 x_1)^2$$

One can deduce :

$$(x_1 x_2 + x_2 x_3 + x_3 x_1)^2 = x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_1^2 + 2(x_1 x_2 + x_2 x_3 + x_3 x_1)$$

So :

$$p^2 = x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_1^2$$

$$y'_1 y'_2 y'_3 = 27q^2 + 3p^2(-2p) + 9p.p^2 + p^3$$

$y'_1 y'_2 y'_3 = 4p^3 + 27q^2$

NOW LET US GENERALIZE SUCH A RESULT for Nth degree EQUATIONS !